Commercial Software, Adware, and Consumer Privacy*

Yossi Spiegel†

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Abstract

The paper studies the choice between selling new software commercially and bundling it with ads and distributing it for free as adware. Adware allows advertisers to send targeted information to specific consumers and may therefore improve their purchasing decisions, but it also entails a loss of privacy. I show that when the (perceived) quality of the software is low, the software provider prefers to distribute it as an adware and collect fees from advertisers. However, as the perceived quality increases he may switch to commercial software. I also study the effect of the choice between commercial software and adware on consumers’ welfare and examine the implications of improvements in the technology of ad banners and the desirability of bans on the use of adware.

JEL Classification: L12, L13, M37

Keywords: commercial software, adware, advertising, display ads, privacy

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†Recanati Graduate School of Business Adminstration, Tel Aviv University, email: spiegel@post.tau.ac.il, http://www.tau.ac.il/~spiegel
1 Introduction

Until the end of the 1990’s, most commercial software was sold to users in retail stores. By the end of the 1990’s, software providers began to distribute their software online. While many software providers require users to pay for the software after a trial period expires, others distribute their software for free as an adware and collect fees from advertisers who use the software to track the behavior of the users and send them targeted information about their products.\(^1\) This paper studies the choice between selling the software commercially to users and distributing it as an adware in the context of a model that takes explicit account of the strategic interaction between software providers, firms that sell consumer products and may advertise them online through display ads, and consumers who buy software and products.

The model considers consumers who differ in their preferences over products, but may not know at the outset which firm sells which product. Display ads allow firms to send consumers targeted information about products that match their tastes and may therefore benefit consumers.\(^2\) At the same time, adware also raises privacy concerns: recent surveys show that many consumers are concerned about the misuse of their private information collected online and refrain from buying online (see e.g., IBM Global Services, 1999).\(^3\) Definitions of privacy vary widely according to context and environment. Posner (1981) discusses several possible definitions, including the “concealment of information,” “peace and quite,” and “freedom and

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\(^1\)This paper considers only “legitimate” ad-supported software which is installed with the end-user consent, but not “spyware” which is often installed without the end-user consent and tracks and collects personal information without consent (see e.g., Urbach and Kibel (2004)). For a discussion on the history of adware, see for example Stern (2004).

\(^2\)There are many types of display ads including banner ads, pop-up ads, and floating ads. In what follows I will simply refer to all of these forms as “display ads.”

\(^3\)Interestingly, the IBM study also shows that in the U.S., Germany, and the U.K. “A minority of consumers ... say they are interested in receiving marketing material. Yet, in far greater numbers, consumers view personalized marketing as a good thing and, in practice, almost half of all consumers in each country have purchased something from a catalog mailed to their home in the past year.” This suggests that there may be a gap between rhetoric and actual behavior.
autonomy.” In this paper I consider the second definition, namely privacy as the right for “peace and quiet.” This right is a main reason behind the “do-not-call list” that is enforced in the U.S. by the FTC and FCC, and is intended to prevent telemarketers from violating consumers’ privacy at home. The desire of consumers for “peace and quiet” is captured in this model by assuming that, in addition to potentially useful information about consumer products, adware users also get a disutility from targeted ads and this disutility increases with the number of ads that they receive. Adware users then face a trade off between the utility from using the software and the beneficial information they get about consumer products and the disutility from privacy loss. In equilibrium, consumers with large privacy concerns do not adopt the adware, while those with relatively small privacy concerns do. The number of adopters in turn determines the willingness of firms to pay for display ads and hence profit from distributing the software as an adware.

The paper shows that in equilibrium, the software will be distributed as an adware provided that its perceived quality is relatively low. When the perceived quality of the software is relatively high, it is more profitable for the software provider to sell the software commercially. This pattern is consistent with the experience of several popular software (e.g., Gozilla and GetRight which are the second and the third most popular download managers on download.com

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4This notion of privacy is also consistent with Smith (2000) who defines privacy as “the desire by each of us for physical space where we can be free of interruption, intrusion, embarrassment, or accountability and the attempt to control the time and manner of disclosures of personal information about ourselves.”

5In a decision from February, 17, 2004, the U.S. Court of Appeals for the Tenth Circuit held that “the do-not-call registry” targets speech that invades the privacy of the home, a personal sanctuary that enjoys a unique status in our constitutional jurisprudence” (Mainstream Marketing Services, Inc., TMG Marketing Inc., and American Teleservices Association v. Federal Trade Commission, et al., U.S. Court of Appeals for the 10th Circuit, No. 03-1429, and consolidated cases). Likewise, in Frisby v. Schultz, 487 U.S. 474, 484 (1988), the Supreme Court of the U.S. held that “One important aspect of residential privacy is protection of the unwilling listener. ... [A] special benefit of the privacy all citizens enjoy within their own walls, which the State may legislate to protect, is an ability to avoid intrusions. Thus, we have repeatedly held that individuals are not required to welcome unwanted speech into their own homes and that the government may protect this freedom.” And, in FCC v. Pacifica Found., 438 U.S. 726, 748 (1978) the Supreme Court of the U.S. held that “[i]n the privacy of the home ... the individuals right to be left alone plainly outweighs the First Amendment rights of an intruder.”
with over 17 million and 11 million downloads respectively) that were first distributed as adware, but then, newer and improved versions were distributed commercially.

Given that adwares are still in their infancy, it is expected that in the near future, the technology of context-based advertising will improve. A case in point is Gmail which is a new free webmail service that Google began to offer in 2004. Gmail places text ads and links to related web pages adjacent to relevant email messages by quickly analyzing their content and determining which ads are most relevant to them.6 Such technological improvements have raised concerns about the increasing loss of privacy on the Internet.7 In our model, such improvements affect both consumers’ privacy as well as their information on consumers’ products. We show that such improvements induce the software provider to offer adware for a wider set of parameters. Hence, consumers with large privacy concerns may be worse off since in order to obtain the software, they are also forced to receive display ads which lower their utility. Yet, our analysis shows that in aggregate, the benefit to consumers from improved information on consumer goods outweighs the associated loss of privacy.

Unlike adware, “spyware” (or even “malware”) is often installed without the end-user’s knowledge and tracks and collects personal information without consent (see e.g., Urbach and Kibel (2004)). The rapid growth of spyware has become a serious problem. Apart from violations of privacy, spyware also causes various technical problems including slow performance of computers, inability to access the Internet, extra icons and pop up ads, Internet or system freezes, and so on.8 This rapid growth has prompted legislators in the U.S. to consider legisla-

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6See http://gmail.google.com/gmail/help/about.html
7For example, a recent article in MSNBC.com Science and Technology, “Privacy advocates target Google mail” http://msnbc.msn.com/id/4679359/ argues, among other things, that Gamil “is spooky enough to rankle privacy advocates, who say Google is going too far by poking through individual e-mails.”
8In a workshop on Spyware held at the FTC in April 2004, Bryson Gordon from McAfee Security, argued that spyware related problems are right now “the single largest issue that we are seeing,” and Maureen Cushman from Dell argued that spyware become ”a huge technical support issue for us,” and that “Spyware related technical support calls have been as high as 12 percent of all technical support requests to the Dell technical support queue.” See http://www.ftc.gov/bcp/workshops/spyware/transcript.pdf
tion that would either ban or substantially restrict the use of ad-supported software. Utah has already passed such legislation that, among other things, prohibits any party from installing software that monitors computer usage, uses context-based triggering mechanisms, and also prohibits the use of context-based pop-ups that obscure the underlying content. Similar legislation is currently pending in California, the U.S. Senate (Spy Block Act, S.2145), the House of Representatives (Safeguard Against Privacy Invasions Act H.R. 2929), and in several other states. Our paper shows that imposing such bans on legitimate ad-supported software may harm consumers since the loss from by not receiving information on consumer goods exceeds the gain from increased privacy.

We also consider competition in the software market between two software providers: one who provides adware and another who provides shareware. Here we consider the equilibrium in the software market when the adware provider sets the price of per impression that firms needs to pay in order to display ads and the commercial software provider sets the retail price of the software. We show that our main conclusions from the single software provider case continue to hold even when there is competition in the software market.

This paper contributes to the small but growing literature on the economics of privacy (see Hui and Png, 2006, for a literature survey). Several papers in this literature have identified the loss of privacy with the disclosure of information on the consumers’ preferences (Acquisti and Varian, 2004; Calzolari and Pavan, 2006; Dodds 2003; Taylor 2002, 2004; and Wathieu, 2002). Such information allows firms to use personalized prices that hurt consumers by extracting their consumers’ surplus. But as Varian (1996) points out, when firms learn information about consumers’ preferences, they can also offer them products that better meet their needs and thereby lower their search costs. Hence, disclosure of information on consumers’ preferences involves a tradeoff between a reduction of search costs and extraction of consumers’ surplus. A different approach to consumers’ loss of privacy is taken by Hann et al. (2008). They consider a game in which firms send costly solicitations to consumers who differ in their willingness to pay
(WTP) for products, while consumers invest efforts in trying to avoid or deflect these solicitations. They show that efforts by low WTP consumers to avoid solicitations make solicitations more cost effective and hence encourage firms to send them. Efforts by high WTP consumers to avoid solicitations have the opposite effect. They also show that solicitation avoidance by low WTP consumers increases direct privacy harm, and that it is optimal to impose a charge on solicitations.\footnote{McAndrews and Morgan study a related model in which phone service users can buy caller ID service in order to block unwanted calls (e.g., from telemarketers) while callers can buy ID blocking which overrides caller ID. They show that without both a well established right to identify callers, and well functioning markets for the services, inefficient allocations emerge.} Hann et al. (2007) empirically examine individuals’ tradeoffs between the benefits and costs of providing personal information to websites. They find that the benefits in terms of monetary rewards and future convenience significantly affect individuals preferences over websites with differing privacy policies. Among U.S. subjects, protection against errors, improper access, and secondary use of personal information is worth $30.49 – $44.62, while among Singapore subjects, it is worth $57.11.

The rest of the paper is organized as follows: Section 2 presents the model. Section 3 characterizes the equilibrium when there is a single software provider who needs to choose whether to sell the new software commercially or distribute it for free as an adaware and then make money by selling ads. Section 4 examines the effect of a technological improvement in the display ads technology on the equilibrium and also considers the policy implications of the model. Section 5 considers the equilibrium when there is competition between commerical software and adware. Concluding remarks are in Section 6.

\section{The model}

There are three types of agents in our model: a software provider, a continuum of consumers (or users), and \( n \geq 2 \) firms that sell consumer products. The software provider has developed new
software and needs to decide whether to sell it commercially to consumers or to bundle it with ads and distribute it for free.\textsuperscript{10} In the latter case, the software provider collects per impression prices from firms that use the software to send consumers targeted information about their products.\textsuperscript{11}

2.1 The timing of the model

The model evolves in three stages. In stage 1, the software provider chooses whether to sell the software commercially, or distribute it for free as an adware. Under the first option, the software provider sets a price, $p^*$, for the software. Under the second option, the software provider sets a per-impression advertising fee, $r$, that firms must pay in order to display ads using the adware. In stage 2, each consumer decides whether or not to get the software. In stage 3, which is reached only if the software provider chooses to distribute the software as an adware, the $n$ firms choose how many ads to display. Finally, consumers buy products from firms.

2.2 Consumers

There is a mass one of potential consumers. Each consumer is interested in getting one software and one out of $n$ consumer products, each of which is produced by a different firm. Consumers belong to $n$ distinct and equally-sized groups: consumers in group $i$ get a utility $s$ if they buy product $i$ and $s - t$ if they buy any other product, where $s \equiv v - p$ is the difference between the gross utility of buying the “right” product, $v$, and the product’s price, $p$. For simplicity, I will assume that $p$ is the same for all products and treat it as an exogenous parameter. Consumers, however, do not necessarily know about all firms at the outset: each consumer in group $i$

\textsuperscript{10}When the programmer chooses to sell the software, he can either sell it as “retail software” through retail stores, or distribute it as shareware in which case users can freely download it from the Internet for a trial period but then need to buy it after the trial period expires.

\textsuperscript{11}An “impression” is a single appearance of an ad on a web page.
learns about product \( i \) only with probability \( \rho \). With probability \( 1 - \rho \), the consumer does not know about product \( i \), but may still end buying it at random with probability \( \frac{1}{n} \). The overall probability that a consumer in group \( i \) buys product \( i \) then is \( \varphi = \rho + \frac{1-\rho}{n} \). With probability \( \varphi \), the consumer buys some other product. The expected utility of a consumer who does not buy software is therefore

\[
\overline{U} = \varphi s + (1 - \varphi) (s - t).
\] (1)

**Users of commercial software:** Using \( q \) to denote the perceived quality of the software, a consumer who buys commercial software gets a net utility \( \theta q - p^* \), where \( \theta \) is the marginal willingness to pay for quality and is drawn from a uniform distribution on the unit interval. Given \( \theta \), the expected utility of each user of commercial software is

\[
U^*(\theta) = \theta q - p^* + \overline{U}.
\] (2)

To ensure that the model attains an interior solution, I will make the following assumption:

**Assumption 1:** \( \frac{q}{2} \leq (1 - \varphi) t \)

Assumption 1 implies that the “average” direct utility from the software, \( \frac{q}{2} \), does not exceed the utility loss due to choosing the “wrong” consumer product. This assumption ensures that in equilibrium, firms will agree to pay for display ads.

**Adware users:** If a consumer adopts an adware, he gets in addition to the software he is interested in, another piece of software that tracks his behavior and enables the software provider to send him targeted ads about products that match his preferences. In the current model, this means that the software provider likes to send adware users in group \( i \) ads about product \( i \). I will assume however that the adware technology is imperfect: the probability that a targeted
ad sent to a user in group $i$ is indeed about product $i$ is only $\phi < 1$. With probability $1 - \phi$, the adware fails to identify the user’s true preferences and hence he receives an ad about a “wrong” product.

Let $k_i$ be the number of impressions that firm $i$ pays for (i.e., the number of times that firm $i$’s ads are displayed on the user’s screen) and let $m \in [0,1]$ be the probability that an ad captures the user’s attention. Assuming that the probability of noticing each impression is independent across impressions, and recalling that each ad is about a relevant product only with probability $\phi$, the overall probability that a consumer in group $i$ notices at least one relevant impression is

$$
\mu_i = 1 - (1 - \phi m)^{k_i}.
$$

(3)

With probability $1 - \mu_i = (1 - \phi m)^{k_i}$, the consumer either ignores all $k_i$ impressions or pays attention only to irrelevant ads. In what follows, I will refer to $\mu_i$ as “consumer attention.”

In what follows it will be easier to express the model in terms of $\mu_i$ rather than $k_i$. To this end, note from equation (3) that

$$
k_i = z \ln (1 - \mu_i), \quad z \equiv \frac{1}{\ln(1 - \phi m)}.
$$

(4)

Equation (4) represents the number of impressions that firm $i$ needs to send in order to ensure attention $\mu_i$ in group $i$ (i.e., probability $\mu_i$ that its product is noticed by adware users in group $i$). Since $\ln (1 - \mu_i) < 0$, $k_i$ decreases with $z$, implying that as $z$ (which is negative) increases towards 0, the ads technology improves, fewer impressions are needed to attract the same level of users’ attention. Hence, $z$ which increases with $m$, serves as a measure of how effective internet ads are in attracting the attention of adware users.

Apart from ads, adware users in group $i$ also learn about product $i$ with probability $\varphi$ (just like users of commercial software and those who do not buy software). Assuming that
the probability of learning about products from an adware is independent of the probability of learning about it from other sources, the probability that an adware user in group $i$ buys product $i$ is $\mu_i + (1 - \mu_i)\varphi$ (the probability that he learns about product $i$ from the adware plus the probability that he does not learn about it from the adware but does learn about it from other sources), while the probability that he buys product $j \neq i$ is $(1 - \mu_i)(1 - \varphi)$. We assume that if the consumer does not learn about product $i$, he learns about one of the $n - 1$ “wrong” products with probability $\frac{1}{n-1}$.

While ads provide adware users with potentially useful information about consumer products, they also violate their privacy by intruding on their right “to be left alone.” We assume that the resulting disutility that adware users bear is directly related to the number of impressions that are displayed on their screens and is given by $\beta k_i$, where $\beta$ is independent of $\theta$ and is uniformly distributed in the population on the interval $[0, B]$. That is, apart from their different marginal willingness to pay for the software, consumers also differ from each other with respect to their disutility from privacy loss. In what follows I shall assume that $B$ is sufficiently large:

**Assumption 2:** $B > \frac{\frac{\varphi}{2} + (1 - \varphi)t}{2z\ln\left(\frac{\frac{\varphi}{2} + (1 - \varphi)t}{2(1 - \varphi)t}\right)}$

This assumption implies that even among consumers with the highest willingness to pay for software (i.e., those with $\theta = 1$), there are some whose privacy concerns are so large that they will not adopt the adware even though it is distributed for free.

Assuming that the perceived quality of the adware is also $q$ and using equation (4), the expected utility of an adware user in group $i$ is given by

$$U_i^a(\theta, \beta) = \theta q - \beta z \ln (1 - \mu_i) + (\mu_i + (1 - \mu_i)\varphi) s + (1 - \mu_i)(1 - \varphi)(s - t)$$

$$= \theta q - \beta z \ln (1 - \mu_i) + \mu_i (1 - \varphi)t + \overline{U}. \quad (5)$$

The third term on the right hand side of equation (5) represents the added information that
the adware user gets about consumer products which increases his chance to find the “right” product. The second term is the disutility from lost privacy. This term decreases with \( z \) since an improvement in the ads’ technology allows firms to send adware users fewer ads in order to get the same level of attention from them. Hence, the privacy of adware users is violated to a lesser extent.

### 2.3 Firms

Firms’ decisions matter in our model only when the software provider offers an adware. Otherwise, each consumer (whether he owns a commercial software or not) either learns about the “right” product with probability \( \varphi \) or else picks one of the \( n - 1 \) “wrong” products at random. By symmetry then, each firm serves a mass \( \frac{1}{n} \) of consumers, each of whom is interested in buying one unit.

Now, suppose that the software provider offers an adware and suppose that a fraction \( \alpha \) of all consumers adopt it. The remaining consumers do not adopt the adware; as in the commercial software case, each firm end ups serving a fraction \( \frac{1}{n} \) of these consumers. Hence, each firm \( i \) faces a mass \( \frac{1 - \alpha}{n} \) of consumers who do not own an adware.

Next, we turn to the demand for firm \( i \)'s product among adware users. The total mass of adware users is \( \alpha \), of which \( \frac{\alpha}{n} \) are in group \( i \).

12 The probability that adware users in group \( i \) buy from firm \( i \) is \( \mu_i + (1 - \mu_i) \varphi \), while the probability that adware users in group \( j \neq i \) buy from firm \( i \) is \( \frac{(1 - \mu_j)(1 - \varphi)}{n - 1} \) (the probability they fail to learn about product \( j \) times the probability they learn about product \( i \) which is one of the \( n - 1 \) “wrong” products). By the law of large numbers, firm \( i \) serves a total of \( \frac{\alpha}{n} \left[ \mu_i + (1 - \mu_i) \varphi + \sum_{j \neq i} \frac{(1 - \mu_j)(1 - \varphi)}{n - 1} \right] \) adware users.

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12 A fraction \( \varphi \) of these consumers are in group \( i \) and buy the product either because they are informed (with probability \( \rho \)) or by chance (with probability \( \frac{1 - \varphi}{n - 1} \)), while a fraction \( 1 - \varphi \) of the consumers are not in group \( i \) and would have been better off buying some other product.
The expected demand that firm $i$ is facing, given the attention levels $\mu_1, \ldots, \mu_n$ is:

$$Q_i(\mu_1, \ldots, \mu_n) = \frac{1 - \alpha}{n} + \frac{\alpha}{n} \left( \mu_i + (1 - \mu_i)\varphi + \sum_{j \neq i} \frac{(1 - \mu_j)(1 - \varphi)}{n - 1} \right).$$  \hspace{1cm} (6)

Assuming that the profit per unit of consumer product is $\pi \equiv p - c$, where $p$ is the products price and $c$ is the (constant) marginal cost, using $r$ to denote the per-impression price that firms pay adware providers, and using equation (4), the expected profit of firm $i$ is

$$\Pi_i(\mu_1, \ldots, \mu_n) = Q_i(\mu_1, \ldots, \mu_n)\pi - \frac{\alpha r z \ln(1 - \mu_i)}{k_i}.$$  \hspace{1cm} (7)

### 2.4 The software provider

For simplicity, the model starts after the software provider has already developed the new software. Hence, the development costs are already sunk and the software provider’s problem is how to distribute the software. If the software provider chooses to sell the software commercially at a price $p^s$, then only consumers with $\theta \geq \frac{p^s}{q}$ will buy it. Since $\theta$ is uniformly distributed on the unit interval, the software provider’s profit is

$$O^s(q, p^s) = p^s \left( 1 - \frac{p^s}{q} \right).$$

The profit maximizing price then is $\hat{p}^s = \frac{q}{2}$ and the resulting profit is $O^s(q, \hat{p}^s) = \frac{q}{4}$.

If the software provider distributes the software for free as an adware he can collects money from firms that use the software to advertise their products. The software provider then sets a price $r$ per impression and lets firms choose how many ads to display. The software provider chooses $r$ to maximize his profit from selling ads; this profit is given by $r \sum_{i=1}^{n} k_i$, or
using (4),
\[ r z \sum_{i=1}^{n} \frac{\alpha}{n} \ln (1 - \mu_i). \]

In stage 1 of the game, the software provider decides whether to sell the software commercially or distribute it as an adware, depending on which option is more profitable.

3 Equilibrium

This section characterizes the subgame perfect equilibrium of the model. The equilibrium in the commercial software subgame is simple: the software provider sells the software at a price of \( \hat{p}^s = \frac{q}{2} \) and serves all consumers with \( \theta \in \left[ \frac{1}{2}, 1 \right] \). The provider’s profit is \( O^s (q, \hat{p}^s) = \frac{q}{4} \). The rest of this section characterizes the equilibrium in the more involved adware case: Sections 3.1 and 3.2 consider stages 2 and 3 in the adware subgame. Section 3.3 considers the software provider’s decision how to distribute the new software by comparing the profits under the commercial software and the adware option.

3.1 Stage 3: the choice of targeted ads

Suppose the software provider chooses to distribute the new software as an adware and sets a price \( r \) per impression. Each firm \( i \) then chooses \( \mu_i \) by maximizing its expected profit. The first order condition for \( \mu_i \) is:

\[
\frac{\partial \Pi_i (\mu_1, \ldots, \mu_n)}{\partial \mu_i} = \frac{\alpha}{n} (1 - \varphi) \pi + \frac{\alpha}{n} \left( \frac{r z}{1 - \mu_i} \right) = 0.
\]

The first term here is the marginal benefit from displayed ads: ads increase the probability that each of the \( \frac{\alpha}{n} (1 - \varphi) \) adware users in group \( i \) who is not already aware of product \( i \) from other sources will learn about it. The profit from selling the product to suc a consumer is \( \pi \). The
second term is the marginal cost of display ads (recall that $z < 0$; hence the second term is negative).

Solving the first order condition for $\mu_i$, reveals that in equilibrium, the attention level in group $i$ given a per-impression price $r$ is

$$
\mu_i = \hat{\mu}(r) \equiv 1 + \frac{r z}{(1 - \varphi) \pi}, \quad i = 1, 2, ..., n. \quad (8)
$$

Assumption 1 ensures that in stage 1 of the model, the software provider will set $r$ such that $\hat{\mu}(r) \geq 0$. If Assumption 1 fails, firms will prefer not to pay for ads.

Recalling that $z < 0$, equation (8) shows that $\hat{\mu}(r)$ is a linearly decreasing function of $r$. Equation (8) shows that $\hat{\mu}(r)$ is increasing with $\pi$ and $z$ but is decreasing with $\varphi$. Intuitively, firms demand more display ads when the market is more profitable ($\pi$ is larger) and when display ads attract consumers’ attention more effectively ($z$ increases towards 0), but demand fewer display ads when adware users are more likely to learn about the “right” products from other sources ($\varphi$ is larger). It is interesting to note that each firm’s demand for display ads is independent of $\alpha$ which is the fraction of consumer who choose to buy adware. The reason for this is that firms pay price per impression, so if there is a smaller number of adware users, their payments to the adware provider decrease proportionally.

### 3.2 Stage 2: consumer’s demand for adware

When the software provider chooses to distribute the software as an adware, each consumer needs to decide whether or not to adopt it. Substituting for $\hat{\mu}(r)$ from (8) into (5) and simplifying, the utility from having an adware is

$$
U^a(\theta, \beta) = \theta q - \beta z \ln (1 - \hat{\mu}(r)) + \hat{\mu}(r) (1 - \varphi) t + U. \quad (9)
$$
The first three terms in this equation capture the extra utility that an adware user gets relative to a non user. The first term, \( \theta q \), is the direct utility from using the software. The second term, \( \beta z \ln (1 - \hat{\mu}(r)) \), is the disutility from privacy loss. The third term is the additional utility from buying the “right” product: without an adware the consumer would buy the “wrong” product with probability \( \hat{\mu}(r)(1 - \varphi) \) and lose utility of \( t \).

Consumers will adopt the adware if and only if \( U_a(\theta, \beta) \geq \overline{U} \). Since \( U_a(\theta, \beta) \) is decreasing with \( \beta \), the equation \( U_a(\theta, \beta) = \overline{U} \) defines, for each value of \( \theta \), a unique value of \( \beta \), denoted \( \hat{\beta}(\theta, q, \hat{\mu}(r)) \), below which the consumer will adopt an adware. Using equation (9),

\[
\hat{\beta}(\theta, q, \hat{\mu}(r)) = \max \left\{ \frac{\theta q + \hat{\mu}(r)(1 - \varphi)t}{z \ln (1 - \hat{\mu}(r))}, B \right\}.
\]

Assumption 2 ensures that in equilibrium, \( \hat{\beta}(\theta, q, \hat{\mu}(r)) < B \), even when \( \theta = 1 \). That is, given the equilibrium number of display ads, some consumers do not adopt the adware due to privacy concerns even if they highly value the software itself (i.e., even if \( \theta = 1 \)). At the other extreme where \( \theta = 0 \), \( \hat{\mu}(r)(1 - \varphi)t/z \ln (1 - \hat{\mu}(r)) \) falls from \( \frac{(1 - \varphi)t}{z} \) when \( \hat{\mu}(r) = 0 \) (by L’Hôpital’s rule, \( \lim_{\hat{\mu}(r)\to0} \frac{\hat{\mu}(r)(1 - \varphi)t}{z \ln (1 - \hat{\mu}(r))} = \frac{(1 - \varphi)t}{z} \)) to 0 when \( \hat{\mu}(r) = 1 \).

Given that \( \hat{\beta}(\theta, q, \hat{\mu}(r)) < B \) for all \( \theta = 1 \), and recalling that \( \theta \) and \( \beta \) are uniformly distributed in the population on the intervals \([0, 1]\) and \([0, B]\), respectively, the mass of consumers who choose to get an adware is

\[
\hat{\alpha}(q, r) = \int_0^1 \frac{\hat{\beta}(\theta, q, \hat{\mu}(r))}{B} d\theta = \frac{\frac{q}{2} + \hat{\mu}(r)(1 - \varphi)t}{Bz \ln (1 - \hat{\mu}(r))}.
\]

Equation (11) shows that the mass of adware adopters is increasing with the quality of the software itself, \( q \), and with the extra utility from buying the “right” product, \( \hat{\mu}(r)(1 - \varphi)t \), but is decreasing with the number of impressions that each adware user receives, \( z \ln (1 - \hat{\mu}(r)) \). Since Assumption 2 guarantees that \( \hat{\beta}(\theta, q, \hat{\mu}(r)) < B \), in equilibrium, \( \hat{\alpha}(q, r) < 1 \).
The aggregate demand of firms for display ads is

\[ Q(q, r) = \alpha(q, r) z \ln (1 - \hat{\mu}(r)), \]

where \( \alpha(q, r) \) is the mass of adware users, and \( k_i = z \ln (1 - \hat{\mu}(r)) \) is the number of impressions sent to each adware user. Using (11),

\[ Q(q, r) = \frac{q^2 + \hat{\mu}(r) (1 - \varphi) t}{B}. \]

Notice that since \( \hat{\mu}'(r) < 0 \), the aggregate demand for ads is a downward sloping function of the per-impression price, \( r \). This result is due to three effects. The first is a “price effect”: an increase in \( r \) induces firms to pay for fewer ads per user (i.e., \( z \ln (1 - \hat{\mu}(r)) \) decreases). Second, there is a “privacy loss effect”: a decrease in the number of ads per user means smaller privacy loss and hence more users wish to adopt an adware. The third effect is an “information effect”: a decrease in the number of ads also means that adware users obtain less information about consumer products and hence are less inclined to adopt the adware. However, the “price effect” and the “privacy loss” effects just cancel each other, so the aggregate demand for ads, which is proportional to the number of adware users, falls with \( r \).

### 3.3 Stage 1: the software provider’s problem

In this stage, the software provider needs to choose whether to commercially sell the new software or distribute it as an adware. Under the first option, the software provider sets a price of \( \hat{p}^s = q \frac{7}{4} \) and earns a profit of \( O^s(q, \hat{p}^s) = \frac{q}{4} \).
Using (13), the software provider’s profit from adware is

\[ O^a(q, r) = r \frac{\left( \frac{q}{2} + \hat{\mu}(r) (1 - \varphi) t \right)}{B} \frac{Q(q, r)}{q(q, r)}. \]  

(14)

Recalling from equation (8) that \( \hat{\mu}(r) \) is linearly decreasing with \( r \), \( O^a(q, r) \) is strictly concave in \( r \). Hence, the profit maximizing price per impression, \( r^* \), is uniquely determined by the the first order condition \( \frac{dO^a(q, r)}{dr} = 0 \). Solving this equation reveals that \( r^* \),

\[ r^* = \frac{-\left( \frac{q}{2} + (1 - \varphi) t \right) \pi}{2zt}. \]  

(15)

Substituting for \( r^* \) from (15) into (8), consumer attention in equilibrium is

\[ \hat{\mu}(r^*) = \frac{(1 - \varphi) t - \frac{q}{2}}{2(1 - \varphi) t}. \]  

(16)

Clearly, \( \hat{\mu}(r^*) < 1 \). Assumption 1 ensures that \( \hat{\mu}(r^*) \geq 0 \). Given \( \hat{\mu}(r^*) \), it is straightforward to verify that Assumption 2 guarantees that \( \hat{\beta}(\theta, q, \hat{\mu}(r^*)) < B \) even when \( \theta = 1 \): some consumers do not adopt an adware due to privacy concerns. This ensures in turn that \( \hat{\alpha}(q, r^*) < 1 \).

Given \( r^* \), the profit from adware is

\[ O^a(q, r^*) = -\frac{\left( \frac{q}{2} + (1 - \varphi) t \right)^2 \pi}{4ztB}. \]  

(17)

Equation (17) shows that \( O^a(q, r^*) \) is increasing with the software’s perceived quality \( q \), with the probability that consumers will pick a “wrong” product when they do not have an adware, \( 1 - \varphi \).

---

\(^{13}\)Notice from (8) that the price per impression cannot exceed \( \bar{r} = \frac{(1 - \varphi) \pi}{z} \), which is the value of \( r \) at which \( \mu(r) = 0 \). Assumption 1 ensures that \( r^* \leq \bar{r} \). When Assumption 1 is violated, the adware provider will set \( r \) just slightly below \( \frac{(1 - \varphi) \pi}{z} \), which is the highest value of \( r \) that still ensures that \( \mu(r) \) is positive.
with the profit per-unit of consumer product, $\pi$, and with $z$ that measures how effective ads are. Intuitively, an increase in $q$ and $1 - \varphi$ makes adware more attractive so more consumers adopt it. Consequently, firms are willing to pay more money to display ads. Likewise, an increase in $\pi$ implies that firms are more eager to advertise. When $z$ increases, firms need to display fewer ads to get consumers’ attention. This boosts the demand for ads and hence the software provider’s profit.

Having found the profit from adware, I now compare it with the profit from commercial software in order to determine the most profitable way to distribute the new software.

**Proposition 1:** The solution to the software provider’s problem is as follows:

(i) If $B < -\frac{2(1-\varphi)\pi}{z}$, the software provider will offer adware for all values of $q$.

(ii) If $B \geq -\frac{2(1-\varphi)\pi}{z}$, the software provider will offer adware if $q \leq q_1$ and will offer commercial software if $q > q_1$, where

$$q_1 \equiv \frac{2t \left(1 + \frac{(1-\varphi)\pi}{zB} - \sqrt{1 + \frac{2(1-\varphi)\pi}{zB}}\right)}{-\frac{\pi}{zB}}.$$  

**Proof:** Suppose that $B < -\frac{2(1-\varphi)\pi}{z}$ and let

$$\Delta(q) \equiv O^a(q, r^*) - O^s(\bar{p}^s) = -\frac{(q + (1-\varphi)\pi t)^2\pi}{4ztB} - \frac{q}{4},$$

be the difference between the profit from adware, $O^a(q, r^*)$, and the profit from commercial software, $O^s(\bar{p}^s)$. Note that $\Delta''(q) = -\frac{\pi}{8ztB} > 0$, so that $\Delta(q)$ has a unique minimum, $q_\text{min}$. Evaluated at $q_\text{min}$,

$$\Delta(q_\text{min}) = \frac{zt}{4\pi} \left[B + \frac{2(1-\varphi)\pi}{z}\right] > 0,$$
where the inequality follows since $z < 0$ and since by assumption, $B < -\frac{2(1-\varphi)\pi}{z}$. Hence when $B < -\frac{2(1-\varphi)\pi}{z}$, $O^a(q, r^*) > O^s(q, \hat{r}^*)$ for all $q$, implying that adware is more profitable.

Next, suppose that $B \geq -\frac{2(1-\varphi)\pi}{z}$. Now $\Delta(q) = 0$ has two solutions:

$$q_1 \equiv \frac{2t}{-\frac{\pi}{zB}} \left(1 + \frac{(1-\varphi)\pi}{zB} - \sqrt{1 + \frac{2(1-\varphi)\pi}{zB}}\right), \quad q_2 \equiv \frac{2t}{-\frac{\pi}{zB}} \left(1 + \frac{(1-\varphi)\pi}{zB} + \sqrt{1 + \frac{2(1-\varphi)\pi}{zB}}\right),$$

where $O^a(q, r^*) > O^s(q, \hat{r}^*)$ if $q < q_1$ or $q > q_2$, and $O^a(q, r^*) < O^s(q, \hat{r}^*)$ if $q_1 \leq q \leq q_2$.

Recalling from Assumption 1 that $q \leq 2(1-\varphi)t$ and noting that $B \geq -\frac{2(1-\varphi)\pi}{z}$ implies that $0 \leq 1 + \frac{2(1-\varphi)t}{zB} \leq 1$, yields

$$q_2 - 2(1-\varphi)t = \frac{2t}{-\frac{\pi}{zB}} \left(1 + \frac{2(1-\varphi)\pi}{zB} + \sqrt{1 + \frac{2(1-\varphi)\pi}{zB}}\right) \geq 0,$$

and

$$q_1 - 2(1-\varphi)t = \frac{2t}{-\frac{\pi}{zB}} \left(1 + \frac{2(1-\varphi)\pi}{zB} - \sqrt{1 + \frac{2(1-\varphi)\pi}{zB}}\right) \leq 0.$$ 

Hence, the feasible solution is $q_1$.  

Proposition 1 is illustrated in Figure 1. When the perceived quality of the software, $q$, is low, selling it commercially is not very profitable. On the other hand, the profit from adware is positive even when $q = 0$ since $O^a(0, r^*) > 0$. Hence, when $q$ is low, the software provider prefers to distribute his software for free as an adware and make money by selling ads. Intuitively, firms will pay for ads only so long as at least some consumers adopt the adware. When $q = 0$, adware users do not benefit from the software itself, but since they do not pay for it, they may still adopt it if the benefit from learning about consumer products via ads exceeds the associated disutility from privacy loss. Equation (9) reveals that for users with relatively small values of $\beta$, this disutility is small relative to the benefits. Consequently, the software provider can make money from adware even when $q = 0$. By continuity, the same is true for small values of $q$.  

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But as $q$ increases, the profit from commercial software may eventually exceed the profit from adware provided that $B \geq -\frac{2(1-\varphi)\pi}{z}$ (when $B < -\frac{2(1-\varphi)\pi}{z}$, the profit from adware exceeds the profit from commercial software for all feasible values of $q$). The reason for this is that the profit from commercial software is directly related and increases linearly with $q$, while the profit from adware is only indirectly affected by $q$ and the slope of $O^a(q, r^*)$ is small when $q$ is not too large.

Casual observation suggests that many popular software are first distributed as adware, but then, newer and improved versions are sold commercially. Examples for this pattern include Gozilla and GetRight which are the second and the third most popular download managers on download.com with over 17 million and 11 million downloads respectively. Proposition 1 provides a possible explanation for this pattern. It should be noted that $q$ need not represent the “true” quality of the software but rather its perceived quality by potential users. If potential users believe that $q$ is lower than it really is and the software provider has no way of crediblyconvincing them otherwise, then it pays the software provider to first distribute the software as an adware. As more consumers use the software and learn about its true quality, the perceived quality of the software increases and the software provider benefits from commercially selling newer versions.

4 Comparative statics and policy implications

As mentioned in the Introduction, adware technology is still relatively new and the technology of sending context-based targeted ads to specific online users is expected to improve further in the near future. Such improvements however have raised concerns about the increasing loss of privacy on the Internet. It is therefore interesting to find out how improvements in adware technology will affect consumers in the context of the current model where both privacy loss as well as the benefits from improved information on consumers’ products are explicitly taken into account.
To address this issue, notice that improvements in adware technology can be either by assuming that the adware technology identifies consumers preferences with greater accuracy and hence sends relevant ads with higher probability (i.e., \( \phi \) is higher) or by assuming that ads capture the attention of users more often (i.e., \( m \) is higher). Either way, the changes pushes \( z = \frac{1}{\ln(1-\phi m)} \) upward towards 0. Hence, I will study the affect of improvements in the adware technology by studying how the equilibrium responds to increases in \( z \).

To this end, recall from (17) that \( O^a(q, r^*) \) increase with \( z \). The reason for this is that an increase in \( z \) boosts the demand of firms for ads and this enables the adware provider to raise the price per impression. Consequently, adware is more profitable than commercial software for a wider set of parameters.

Does an increase in \( z \) benefit consumers as well? To address this question, recall that consumers adopt an adware if and only if \( \beta \leq \beta^1(\theta, q, \mu(r^*)) \); the utility of each adware user in group \( i \) is \( U^a_i(\theta, \beta) \), while the utility of each non user is \( \overline{U} \). Recalling that there are \( \frac{1}{n} \) consumers in each group and using equations (5) and (10), consumer surplus is given by

\[
CS^a(q, r) = \sum_{i=1}^{n} \frac{1}{n} \int_0^1 \left[ \int_0^{\beta^1(\theta, q, \mu(r^*))} \frac{U^a_i(\theta, \beta)}{B} \, d\beta + \int_{\beta^1(\theta, q, \mu(r^*))}^{\beta^2} \frac{\overline{U}}{B} \, d\beta \right] \, d\theta
\]

(18)

Substituting for \( \hat{\mu}(r^*) \) from (16) into (18), the value of consumer surplus in equilibrium is

\[
CS^a(q, r^*) = \frac{3 \left( \frac{q}{2} + (1-\varphi) \right)^2 + 4 \left( \frac{q}{2} \right)^2}{24 z B \ln \left( \frac{2+(1-\varphi)/t}{2(1-\varphi)t} \right)} + \overline{U}.
\]

(19)

By Assumption 1, \( \ln \left( \frac{2+(1-\varphi)/t}{2(1-\varphi)t} \right) \leq 0 \), implying that \( CS^a(q, r^*) \) is increasing with \( z \). Hence, technological improvements in the effectiveness of targeted display ads benefits consumers. In-
tuitively, an increase in $z$ affects consumers in two ways. First, holding consumer attention $\hat{\mu}(r)$ fixed, an increase in $z$ means that fewer impressions are needed to send relevant information to adware users. Hence, adware entails a smaller loss of privacy. Second, an increase in $z$ affects $\hat{\mu}(r)$ itself both directly, as well as indirectly through its effect on $r^\ast$. The direct effect of $z$ on $\hat{\mu}(r)$ is positive because an increase in $z$ makes display ads a more effective marketing tool. At the same time, an increase in $z$ induces the software provider to raise $r^\ast$ and this depresses the demand for display ads. It turns out that the direct and indirect effects cancel each other out so $\hat{\mu}(r^\ast)$ is independent of $z$. Consequently, only the first, positive, effect is at work, so an increase in $z$ makes adware more attractive to consumers. Since consumers can always choose not to adopt the adware, their surplus must increase.

It should also be noted that since $\hat{\mu}(r^\ast)$ is independent of $z$, the number of impressions that each adware user receives in equilibrium, $z \ln (1 - \hat{\mu}(r^\ast))$, is decreasing with $z$. This implies in turn that an improvement in the technology of display ads will lead to less violation of privacy rather than more as some technological experts argue.

We summarize this discussion in the following proposition:

**Proposition 2:** Following an improvement in the technology of display ads that increases $z$ (either due to an increase in the accuracy of identifying the consumer’s preferences or in the probability of attracting his attention):

(i) the software provider adopts adware for a larger set of parameters and raises the price of display ads,

(ii) more consumers adopt the adware and both the software provider and consumers become better off,

(iii) fewer impression are sent in equilibrium so adware users face smaller loss of privacy.
I now proceed to evaluate the policy implications of Proposition 1. I begin by asking the following question: suppose the software provider has decided to distribute the software as an adware. Is the price per impression that he sets and the resulting equilibrium number of impressions that each adware user receives optimal? To address this question, I first need to characterize the socially efficient outcome. As usual, the measure of social welfare is the sum of the consumers’ surplus plus profits; in the current context, profits include the profits of firms and the profit of the adware provider.

Given some price per impression, $r$, consumers’ surplus under adware is given by (18), the profit of the adware provider is given by (14), and the aggregate profit of firms is $\sum_{i=1}^{n} \Pi_i(\hat{\mu}(r), \ldots, \hat{\mu}(r))$. Since the payments of firms to the adware provider wash out, and noting that the aggregate demand for consumer products is $\sum_{i=1}^{n} Q_i(\hat{\mu}(r), \ldots, \hat{\mu}(r))$, the aggregate profit of firms and the adware provider is $\pi$. Hence, given $r$, social welfare in the adware’s case is

$$W^a(q, r) = \frac{3}{6zB} \ln \left(1 - \frac{q}{2(1 - \varphi)t}\right) + \frac{q}{2} + U + \pi.$$ (20)

Let $r^{**}$ be the socially optimal per impression price that maximizes $W^a(q, r)$. Since $W^a(q, r)$ differs from $CS^a(q, r)$ only by a constant, $r^{**}$ also maximizes consumer surplus. Now, notice that as $\hat{\mu}(r)$ goes to 0, the denominator of $W^a(q, r)$ goes to 0, while the numerator goes to $q^2$. Hence, consumers benefit from a high price per-impression that lowers $\hat{\mu}(r)$ towards 0.

Since in equilibrium, $\hat{\mu}(r) = \frac{(1 - \varphi)t - \frac{q}{2}}{2(1 - \varphi)t} > 0$, I can report the following result:

**Proposition 3:** In equilibrium there is an excessive number of display ads relative to the number that maximizes consumer surplus and is socially optimal.

Proposition 3 indicates that whenever the software provider offers adware, the equilibrium price per impression, $r$, is excessive; as a result, there is too little usage of display ads by firms.\(^{14}\)

\(^{14}\)It is worth emphasizing again that I only consider “legitimate” ad-supported software and not spyware or...
Intuitively, when the software provider sets \( r \), he fails to take into account the resulting effect on adware users. Given that an increase in \( r \) induces firms to buy fewer display ads, it is clear that \( r \) will end up being excessive if on average, the benefit of adware users from information on consumers’ products exceeds their disutility from privacy loss. For adware users with low values of \( \beta \), the benefit clearly exceeds the cost. For adware users with high values of \( \beta \), the cost exceeds the benefit since adware users also get a utility \( q \) from using the software itself and hence are willing to adopt the adware even when their disutility from privacy loss exceeds the benefit from information about consumers’ products. As \( q \) increases, there are more adware users for whom the net effect of display ads is negative. But since the software provider switches to commercial software when \( q \) increases above \( q_1 \), the fraction of adware users for whom the effect of display ads is negative must be limited. Proposition 3 shows that, indeed, so long as the software providers offers an adware, the aggregate benefit of consumers from display ads exceeds the associated disutility. The software provider fails to take into account this net benefit and hence sets an excessive price for display ads.

Proposition 3 compares the equilibrium value of \( r \) with its socially optimal value. However, this comparison is relevant only if the software provider distributes his software as an adware. Our next task is to compare the equilibrium choice between adware and commercial software with the socially optimal choice.

In the commercial software case, only consumers with \( \theta \geq \frac{p_s}{q} \) buy the software and obtain utility of \( U^s(\theta) \) each (see (2)). Consumers with \( \theta < \frac{p_s}{q} \) do not buy the software and their utility is \( U \). Recalling that in equilibrium, \( \hat{p}^s = \frac{q}{2} \), consumer surplus under commercial software is

\[
CS^s(q, \hat{p}^s) = \int_0^{\frac{q}{2}} Ud\theta + \int_{\frac{q}{2}}^1 U^s(\theta)d\theta = \frac{q}{8} + U.
\]
Recalling that the profit from commercial software is \( O^* (q, \hat{p}^*) = \frac{q}{4} \) and the aggregate profits of firms is \( \pi \), social welfare in the commercial software case is

\[
W^* (q, \hat{p}^*) = \frac{3q}{8} + \mathcal{U} + \pi.
\] (21)

**Proposition 4:** The software provider offers an adware for a smaller set of parameters than is socially efficient whenever \( r^* z \ln (1 - \hat{\mu}(r^*)) \leq \frac{1}{2} \) and offers an adware for a wider set of parameters than is socially efficient whenever \( r^{**} z \ln (1 - \hat{\mu}(r^{**})) \geq \frac{1}{2} \).

**Proof:** Recalling that \( O^*(q, p^*) = \frac{q}{4} \) and using (14), the software provider will distribute his software as an adware if and only if

\[
\frac{r^* (\frac{q}{4} + \hat{\mu} (r^*) (1 - \varphi) t) - \hat{\mu}(r^*) B}{B} \geq \frac{q}{4}.
\] (22)

It is socially efficient to distribute the software as an adware if and only if \( W^a(q, r^{**}) \geq W^* (q, \hat{p}^*) \). Using equations (20) and (21), this condition is equivalent to

\[
\frac{(\frac{q}{2} + \hat{\mu} (r^{**}) (1 - \varphi) t)^2 + \frac{1}{3} (\frac{q}{2})^2}{3zB \ln (1 - \hat{\mu}(r^{**}))} \geq \frac{q}{4}.
\] (23)

Hence, to prove the proposition it is sufficient to prove that the left side of (23) exceeds the left side of (22). Now, suppose that \( r^* z \ln (1 - \hat{\mu}(r^*)) \leq \frac{1}{2} \). Then,

\[
\frac{(\frac{q}{2} + \hat{\mu} (r^{**}) (1 - \varphi) t)^2 + \frac{1}{3} (\frac{q}{2})^2}{3zB \ln (1 - \hat{\mu}(r^{**}))} \geq \frac{(\frac{q}{2} + \hat{\mu} (r^*) (1 - \varphi) t)^2 + \frac{1}{3} (\frac{q}{2})^2}{3zB \ln (1 - \hat{\mu}(r^*))}
\]

\[
\geq \frac{r^* (\frac{q}{2} + \hat{\mu} (r^*) (1 - \varphi) t)}{B}
\]
\[
\frac{(q + \hat{\mu}(r^*) (1 - \varphi) t)^2}{2zB \ln (1 - \hat{\mu}(r^*)))} \geq \frac{(q + \hat{\mu}(r^*) (1 - \varphi) t)^2}{2zB \ln (1 - \hat{\mu}(r^*)))} \\
\geq \frac{(q + \hat{\mu}(r^*) (1 - \varphi) t)^2}{2zB \ln (1 - \hat{\mu}(r^*)))} \\
\geq \frac{(q + \hat{\mu}(r^*) (1 - \varphi) t)^2 r^*}{B},
\]

where the first inequality follows by revealed preferences \((r^* \text{ maximizes the left side of (23))}.\)

On the other hand, if \(r^* z \ln (1 - \hat{\mu}(r^*)) \geq \frac{1}{2}, \text{ then,}\)

\[
\frac{(q + \hat{\mu}(r^*) (1 - \varphi) t)^2 r^*}{B} \geq \frac{(q + \hat{\mu}(r^*) (1 - \varphi) t)^2 r^*}{B} \\
\geq \frac{(q + \hat{\mu}(r^*) (1 - \varphi) t)^2}{2zB \ln (1 - \hat{\mu}(r^*)))},
\]

where the first inequality follows by revealed preferences \((r^* \text{ maximizes the left side of (22))}.\)

\[\blacksquare\]

To interpret Proposition 4, recall from (12) that \(z \ln (1 - \hat{\mu}(r))\) is the aggregate demand for display ads when all consumers adopt the adware. Hence, \(r z \ln (1 - \hat{\mu}(r))\) is the software provider’s profit when the adware market is covered. Proposition 4 states that the software provider will underprovide adware if, evaluated at the profit maximizing price of display ads, \(r^*\), this profit is sufficiently small, but will overprovide adware if at the socially optimal price of display ads, \(r^{**}\), this profit is sufficiently large. To go deeper into the sufficient conditions in Proposition 4, note that

\[
r^*z \ln (1 - \hat{\mu}(r^*)) = -\frac{(q + (1 - \varphi) t) \pi}{2t} \ln \left( \frac{q + (1 - \varphi) t}{2 (1 - \varphi) t} \right).
\]

By Assumption 1, \(\ln \left( \frac{q + (1 - \varphi) t}{2 (1 - \varphi) t} \right) \leq 0\), so this expression increases with \(\pi\). Hence, underprovision of adware is especially likely when the profit of firms on consumers’ products, \(\pi\), is small. By
contrast,  
\[ r^{**} z \ln (1 - \hat{\mu}(r^{**})) = r^{**} z \ln \left(1 - \frac{r^{**} z}{(1 - \varphi) \pi}\right), \]
where \( r^{**} \) maximizes (20) and hence it is independent of \( \pi \). Hence, \( r^{**} z \ln (1 - \hat{\mu}(r^{**})) \) is decreasing with \( \pi \), implying that the sufficient condition for overprovision of adware is especially likely to hold when \( \pi \) is large.

As we mentioned in the Introduction, currently, U.S. legislators are considering legislation that would regulate or even completely ban any software that monitors usage of the internet and transmits information back from a location. Such legislation is mainly motivated by the rapid growth of spyware and malware. However, the legislation may also effectively make it impossible to distribute legitimate adware on line. Using our model we can examine the desirability of bans on adware.

**Proposition 5:** *A ban on adware hurts both consumers and the software provider when \( q \leq q_1 \) and is irrelevant otherwise.*

**Proof:** When \( q > q_1 \), the software provider offers commercial software, so a ban on adware is irrelevant. When \( q \leq q_1 \), the software provider prefers to offer adware. By revealed preferences, it is obvious that a ban on adware would hurt the software provider in this range. As for consumers, note that consumers’ surplus is \( \overline{U} \) in the commercial software case and \( CS(q, r^*) \) in the adware case. The difference between the two expressions is

\[
\overline{U} - CS(q, r^*) = -\frac{(q + (1 - \varphi) t)^2}{8 z B \ln \left(\frac{q + (1 - \varphi) t}{2(1 - \varphi) r^*}\right)} < 0.
\]

Hence, a ban on adware hurts consumers as well. ■

That a ban on adware hurts the software provider is obvious. Less obvious is why such
a ban hurts consumers. The reason for this is simple however: when the software provider offers commercial software, he charges consumers a price equal to their benefit from using the software. Hence, consumers get no surplus from commercial software. On the other hand, when the software provider offers adware, consumers with low privacy concerns get a positive surplus from the adware while those with high privacy concerns do not adopt it. Hence, a ban on adware hurts consumers.

5 Competition in the software market

In this section I consider competition between two software providers who develop software of equal quality. Given that the quality of the software is the same, it is clear that in order to avoid Bertrand competition, one software provider will choose to commercially sell the software while the other will choose to distribute it as adware. In what follows, we study the ensuing competition between the two software providers by solving the following three stage game. In stage 1, the two software providers simultaneously set prices: the software provider who sells the software commercially sets the software’s price $p^*$, while adware software provider sets the per-impression price, $r$. Given $r$, firms decide in stage 2 of the game how many ads to pay for. In stage 3, each consumer decides whether to buy a commercial software or adopt an adware. Finally, consumers buy products and all payoffs are realized.

The profit from selling the commercial software is

$$(1 - \alpha) p^*,$$

where $1 - \alpha$ is the fraction of consumers who decide to purchase commercial software. The profit
of the adware software provider is

\[ rz \sum_{i=1}^{n} \frac{\alpha}{n} \ln (1 - \mu_i) \].

Given \( r \), the intensity of advertising through display ads, \( \hat{\mu}(r) \), is given by equation (8), just as in the single software provider’s case. Notice that each firm \( i \)'s decision about \( \mu_i \) is independent of the number of consumers who will eventually get an adware. Hence, we can solve for the second stage of the game independently of the third stage. The equilibrium prices will be denoted by \( \hat{p}^s \) and \( \hat{r} \).

5.1 Commercial software vs. adware

Consumers decide whether to buy commercial software or get an adware by comparing their utilities from the two options. Using equations (2) and (9), and noting that \( \mu_i = \hat{\mu}(r) \) for all \( i \), it follows that consumers will choose to get adware if

\[ U^a_i(\beta) - U^s = p^s - \beta z \ln (1 - \hat{\mu}(r)) + \hat{\mu}(r) (1 - \varphi) t \geq 0, \]

and will choose to buy commercial software otherwise. Since \( \ln (1 - \hat{\mu}(r)) \leq 0 \) and \( z < 0 \), this expression is decreasing with \( \beta \). Hence, the equation \( U^a(\beta) = U^s \) defines a unique value of \( \beta \), denoted \( \hat{\beta}(p^s, \hat{\mu}(r)) \), below which the consumer will choose to get an adware and above which he will choose to buy the commercial software:

\[ \hat{\beta}(p^s, \hat{\mu}(r)) = \frac{p^s + \hat{\mu}(r) (1 - \varphi) t}{z \ln (1 - \hat{\mu}(r))}. \] \hspace{1cm} (24)

That is, consumers with small privacy concerns will adopt the adware while those with large concerns will purchase the commercial software. Notice that \( \hat{\beta}(p^s, \hat{\mu}(r)) \) is an increasing func-
tion of $p^s$: the higher the price of the commercial software, the more consumers will adopt a
adware rather than commercial software. In other words, adware and commercial software are
substitutes for consumers.

In order to avoid corner solutions, we shall make the following assumption:

Assumption 3: $B > \hat{\beta}(\hat{p}^s, \hat{\mu}(\hat{r}))$.

Assumption 3 modifies Assumption 2 by setting a new lower bound on $B$. This assump-
tion ensures that $B$ is sufficiently large to ensure that in equilibrium, at least some consumers
will find the disutility from privacy loss so large that they will prefer to buy commercial software
instead of adware. Of course, the commercial software software provider can always lower $p^s$ to
induce some consumers to buy the software. Assumption 3 ensures that if $p^s$ is lowered to 0,
the software provider can attract at least some consumers.

Since $\beta$ is uniformly distributed in the population on the interval $[0, B]$, the mass of
consumers who adopt an adware is

$$\hat{\alpha}(p^s, r) = \frac{\beta(p^s, \hat{\mu}(r))}{B}. \quad (25)$$

Given $\hat{\alpha}(p^s, r)$, the demand for commercial software is $1 - \hat{\alpha}(p^s, r)$, so the profit from selling the
commercial software is

$$O^c(p^s, r) = (1 - \hat{\alpha}(p^s, r)) p^s = \left(1 - \frac{p^s + \hat{\mu}(r) (1 - \varphi) t}{z B \ln (1 - \hat{\mu}(r))}\right) p^s, \quad (26)$$

where $\hat{\mu}(r)$ is given by equation (8). The profit of the adware software provider is

$$O^a(p^s, r) = \hat{\alpha}(p^s, r) z \ln (1 - \hat{\mu}(r)) r = \frac{(p^s + \hat{\mu}(r) (1 - \varphi) t) r}{B}. \quad (27)$$
5.2 Prices of commercial software and per-impression prices

In the first stage of the game, the software provider who sells the commercial software sets its price, $p^s$, while the adware software provider sets the per-impression price, $r$.

**Lemma 1:** There exists a unique Nash equilibrium, $(\tilde{p}^s, \tilde{r})$, where $\tilde{p}^s > 0$ and $-\frac{(1-\varphi)\pi}{3z} < \tilde{r} < -\frac{(1-\varphi)\pi}{2z}$.

**Proof:** It is straightforward to verify that $\frac{\partial^2 O^s(p^s, r)}{\partial (p^s)^2} = -z B \ln(1-\mu(r)) < 0$ and $\frac{\partial^2 O^a(p^s, r)}{\partial r^2} = -\frac{2}{z \pi} < 0$. Hence, the best response functions of the two software providers are uniquely defined by the first order conditions, $\frac{\partial O^s(p^s, r)}{\partial p^s} = 0$ and $\frac{\partial O^a(p^s, r)}{\partial r} = 0$. Using equation (8) and simplifying, the best response functions are given by:

$$BR^s(r) = \frac{z B \ln \left( \frac{-rz}{(1-\varphi)\pi} \right) - (1 - \varphi) t - \frac{rzt}{\pi}}{2},$$

(28)

and

$$BR^a(p^s) = -\frac{(p^s + (1 - \varphi) t) \pi}{2zt}.$$  

(29)

To prove the existence of a Nash equilibrium, we must prove that (28) and (29) intersect in the $(\tilde{p}^s, \tilde{r})$ space. Substituting for $p^s = BR^s(r)$ from (28) into (29) yields

$$\frac{3rt}{\pi} + B \ln \left( \frac{-rz}{(1-\varphi)\pi} \frac{1 - \varphi}{z} \right) = \frac{(1 - \varphi) t}{-z}.$$  

(30)

The left hand side of (30) increases with $r$, while the right hand side is independent of $r$. Moreover, as $r$ goes to 0, the left hand side of (30) goes to $-\infty$ while as $r$ goes to $\infty$, the hand side of the right hand side of (30) goes to $\infty$. Hence, equation (30) has a unique solution that we denote by $\tilde{r}$. 

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By (29), it follows that in equilibrium,

\[ \hat{p}^* = -\frac{2\hat{r}zt}{\pi} - (1 - \varphi) t. \] (31)

Since \( \hat{r} \) is unique, so is \( \hat{p}^* \). Moreover, substituting for \( \ln \left( \frac{-\hat{r}z}{(1-\varphi)\pi} \right) \) from (30) into \( \hat{\beta} (\hat{p}^*, \hat{\mu} (\hat{r})) \) and simplifying terms, reveals that Assumption 3 ensures that \( -\frac{2\hat{r}zt}{\pi} - (1 - \varphi) t > 0 \). Hence, (31) implies that \( \hat{p}^* > 0 \).

Finally, \( -\frac{2\hat{r}zt}{\pi} - (1 - \varphi) t > 0 \) implies that

\[ \hat{r} < -\frac{(1 - \varphi) \pi}{2z}. \]

On the other hand, since \( \ln \left( \frac{-\hat{r}z}{(1-\varphi)\pi} \right) < 0 \), (30) implies that \( \frac{3\hat{r}}{\pi} + \frac{(1-\varphi)}{z} > 0 \), or

\[ \hat{r} > -\frac{(1 - \varphi) \pi}{3z}. \]

Next we examine how an improvement in the display ads’ technology that increases \( z \) towards 0 affects the equilibrium prices, profits, and consumers’ surplus.

**Proposition 6:** An increase in \( z \) leads to an increase in \( \hat{r} \) and a decrease in \( \hat{p}^* \). Moreover, \( O^*(\hat{p}^*, \hat{r}) \) decreases while \( O^*(\hat{p}^*, \hat{r}) \) and consumers’ surplus increase.

**Proof:** \( \hat{r} \) is defined implicitly by (30). Differentiating this equation with respect to \( \hat{r} \) and \( z \) and rearranging terms,

\[ \frac{\partial \hat{r}}{\partial z} = \frac{(1 - \varphi) t - zB}{z^2 \left( \frac{3t}{\pi} + \frac{B}{\hat{r}} \right)} > 0. \] (32)
Differentiating (31) with respect to \( \hat{p}^s \) and \( z \), using (32), and rearranging terms,

\[
\frac{\partial \hat{p}^s}{\partial z} = \frac{-2t}{\pi} \left( \hat{r} + z \frac{\partial \hat{r}}{\partial z} \right) = \frac{-2t}{\pi} \left( \frac{3\hat{r}t}{\pi} + \frac{(1-\varphi)t}{z} \right) < 0, \tag{33}
\]

where the last equality follows because by (30), \( \frac{3\hat{r}t}{\pi} + \frac{(1-\varphi)t}{z} = -B \ln \left( \frac{-\hat{r}z}{(1-\varphi)\pi} \right) > 0 \).

Differentiating (27) with respect to \( z \), using the envelope theorem, and substituting from (33) for \( \frac{\partial \hat{p}^s}{\partial z} \):

\[
\frac{\partial O^s(\hat{p}^s, \hat{r})}{\partial z} = \frac{\hat{r}^2 t}{\pi B} + \frac{\hat{r}}{B} \frac{\partial \hat{p}^s}{\partial z} = \frac{\hat{r}t}{\pi B \left( \frac{3\hat{r}t}{\pi} + \frac{B}{\hat{r}} \right)} \left[ B - \frac{3\hat{r}t}{\pi} - \frac{2(1-\varphi)t}{z} \right] > \frac{\hat{r}t}{\pi B \left( \frac{3\hat{r}t}{\pi} + \frac{B}{\hat{r}} \right)} \left[ B - \frac{(1-\varphi)t}{2z} \right] > 0,
\]

where the first inequality follows because by Lemma 1, \( \hat{r} < -\frac{(1-\varphi)\pi}{2z} \), and the second inequality follows because \( z < 0 \).

Differentiating (26) with respect to \( z \), using the envelope theorem, substituting for \( \hat{p}^s \) from (31), and rearranging,

\[
\frac{\partial O^s(\hat{p}^s, \hat{r})}{\partial z} = -\frac{t}{\pi} \left( 2\hat{r} + z \frac{\partial \hat{r}}{\partial z} \right) \ln \left( \frac{-\hat{r}z}{(1-\varphi)\pi} \right) \hat{p}^s + z \frac{\partial \hat{r}}{\partial z} \hat{p}^s \\
\frac{zB \left( \ln \left( \frac{-\hat{r}z}{(1-\varphi)\pi} \right) \right)^2}{\hat{p}^s}.
\]

Since \( z < 0 \),

\[
\text{sign} \left( \frac{\partial O^s(\hat{p}^s, \hat{r})}{\partial z} \right) = \text{sign} \left( \left( 2\hat{r} + z \frac{\partial \hat{r}}{\partial z} \right) \ln \left( \frac{-\hat{r}z}{(1-\varphi)\pi} \right) + \hat{r} + z \frac{\partial \hat{r}}{\partial z} \right).
\]

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Substituting for $\ln \left( \frac{-r_z}{(1-\varphi)\pi} \right)$ from (30) and for $\frac{\partial \hat{\mu}}{\partial z}$ from (32) and rearranging terms,

$$
\text{sign} \left( \frac{\partial \Omega^s(\hat{p}^s, \hat{r})}{\partial z} \right) = \text{sign} \left[ 2\hat{r} + \frac{(1-\varphi) t - zB}{z \left( \frac{3\pi}{\pi} + \frac{B}{r} \right)} \right] \left( -3\frac{\hat{r}t}{\pi B} - \frac{(1-\varphi) t}{zB} \right) + \hat{r} + \frac{(1-\varphi) t - zB}{z \left( \frac{3\pi}{\pi} + \frac{B}{r} \right)} 
$$

$$
= \text{sign} \left[ -\frac{6\hat{r}t}{\pi} + \frac{(1-\varphi) t}{z} \left( 3\frac{\hat{r}t}{\pi} + (1-\varphi) t \right) \right]. 
$$

The proof is completed by noting that the bracketed expression is negative since equation (30) implies that $\frac{3\hat{r}t}{\pi} + \frac{(1-\varphi) t}{z} = -B \ln \left( \frac{-r_z}{(1-\varphi)\pi} \right) > 0$ and $\frac{6\hat{r}t}{\pi} + \frac{(1-\varphi) t}{z} = \frac{3\hat{r}t}{\pi} - B \ln \left( \frac{-r_z}{(1-\varphi)\pi} \right) > 0$.

To examine the effect of $z$ on consumers, recall that consumers get an adware if $\beta \leq \hat{\beta}(\hat{p}^s, \hat{\mu}(\hat{r}))$ and commercial software if $\beta > \hat{\beta}(\hat{p}^s, \hat{\mu}(\hat{r}))$. The utility of each adware user in group $i$ is $U^a_i(\beta)$ while the utility of each user of commercial software is $U^s$. Using equations (2), (9), (24), (8), and (31), it follows that consumers’ surplus is given by

$$
CS(\hat{p}^s, \hat{r}) = \sum_{i=1}^{n} \frac{1}{n} \int_{0}^{\beta(\hat{p}^s, \hat{\mu}(\hat{r}))} \frac{U^a_i(\beta)}{B} d\beta + \int_{\beta(\hat{p}^s, \hat{\mu}(\hat{r}))}^{\hat{r}^s} \frac{U^s}{B} d\beta 
$$

$$
= \frac{(\hat{p}^s + \hat{\mu}(\hat{r})(1-\varphi) t)^2}{2zB \ln \left( 1 - \frac{1-\mu(\hat{r})}{(1-\varphi)\pi} \right)} + q - \hat{p}^s + \bar{U} 
$$

$$
= \frac{\hat{r}^2 z t^2}{2\pi^2 B \ln \left( \frac{-r_z}{(1-\varphi)\pi} \right)} + q - \hat{p}^s + \bar{U}.
$$

Differentiating (34) with respect to $z$,

$$
\frac{\partial CS(\hat{p}^s, \hat{r})}{\partial z} = \frac{\hat{r}^2 t^2}{2\pi^2 B \ln \left( \frac{-r_z}{(1-\varphi)\pi} \right)^2} \left[ \ln \left( \frac{-\hat{r}z}{(1-\varphi)\pi} \right) \left( \hat{r} + z \frac{\partial \hat{r}}{\partial z} \right) - \left( \hat{r} + \frac{\partial \hat{r}}{\partial z} \right) \right] - \frac{\partial \hat{p}^s}{\partial z}.
$$
Substituting for \( \ln \left( \frac{-r_z}{(1-\varphi)\pi} \right) \) from (30) and for \( \frac{\partial p^s}{\partial z} \) from (32) and rearranging terms

\[
\frac{\partial CS(\hat{p}^s, \hat{r})}{\partial z} = \frac{\hat{r} t^2 G(\hat{r})}{2 \pi^3 z^2 \left( \frac{3\pi t}{\pi} + B \right) \left( \frac{3\pi}{\pi} + \frac{(1-\varphi)}{z} \right)},
\]

where

\[
G(\hat{r}) \equiv 4(1-\varphi)^2 \pi^2 + 22(1-\varphi)\pi \hat{r} z + 33\hat{r}^2 z^2.
\]

The denominator in \( \frac{\partial CS(\hat{p}^s, \hat{r})}{\partial z} \) is positive since equation (30) implies that \( \frac{2\hat{r}}{\pi} + \frac{(1-\varphi)}{\pi} = -\frac{B}{t} \ln \left( \frac{-r_z}{(1-\varphi)\pi} \right) > 0 \). To determine the sign of \( G(\hat{r}) \), note that \( G(\hat{r}) \) is convex function of \( \hat{r} \) and attains a unique minimum point at \( \hat{r} = \frac{(1-\varphi)^2 \pi^2}{3} \). Since \( G \left( \frac{(1-\varphi)^2 \pi^2}{3} \right) = 66z^2 > 0 \), it follows that the \( G(\hat{r}) > 0 \) for all \( \hat{r} \). Hence, \( \frac{\partial CS(\hat{p}^s, \hat{r})}{\partial z} > 0 \).

Proposition 6 shows that the conclusions from Section 4 about the impact of an improvement in the technology of display ads carry over to the case where there is competition in the software market. In particular, the technological improvement benefits the adware software provider as well as consumers. Proposition 6 shows that the improvement lowers the price of the commercial software and hurts the software provider who sells this software.

Finally, we reexamine the effect of a ban on the distribution of adware on the software providers and on consumers when there is competition in the software market.

**Proposition 7:** A ban on adware hurts both consumers and the software providers.

**Proof:** A ban on adware induces both software providers to sell their software commercially. Since the software of both software providers is assumed to have the same quality, competition in the software market drives \( p^s \) to 0. Hence, consumers’ surplus is \( q + U \). Absent a ban, consumers’ surplus is \( CS(\hat{p}^s, \hat{r}) \). Substituting for \( \hat{p}^s \) from (31) and for \( B \ln \left( \frac{-r_z}{(1-\varphi)\pi} \right) \) from (30)
and simplifying, the difference between the two expressions is

\[
q + U - CS(\hat{p}, \hat{r}) = \frac{\hat{r}^2 z t^2}{2\pi^2 B \ln \left( \frac{-\hat{r} z}{(1-\varphi) \pi} \right)} - \hat{p}^s
\]

\[
= - \frac{tM(\hat{r})}{2\pi^2 z \left( \frac{3\varphi}{\pi} + \frac{(1-\varphi)}{z} \right)},
\]

where

\[
M(\hat{r}) = 11\hat{r}^2 z^2 + 10\hat{r} z (1 - \varphi) \pi + 2 (1 - \varphi)^2 \pi^2.
\]

The sign of \(q + U - CS(\hat{p}, \hat{r})\) depends on the sign of \(M(\hat{r})\) since \(30\) implies that \(\frac{3\varphi}{\pi} + \frac{(1-\varphi)}{z} = -\frac{B}{\pi} \ln \left( \frac{-\hat{r} z}{(1-\varphi) \pi} \right) > 0\). \(M(\hat{r})\) is a convex function of \(\hat{r}\) and \(M(\hat{r}) = 0\) has two solutions, \(\hat{r}_1 = -0.297(1-\varphi)\pi\) and \(\hat{r}_2 = -0.612(1-\varphi)\pi\) such that a ban on adware harms consumers if and only if \(\hat{r}_1 \leq \hat{r} \leq \hat{r}_2\). Given Lemma 1, \(\hat{r}\) is indeed in this range. \(\blacksquare\)

Propositions 7 shows that the welfare implications of a ban on adware derived in Section 4 are robust to the introduction of competition in the software market: the ban harms the software providers as well as consumers.

6 Conclusion

We have developed a framework for studying the choice of software providers between selling their software commercially and distributing it for free as adware and making money by selling ads. Our model takes explicit account of the strategic interaction between the software providers, advertisers, and consumers and highlights the tradeoff that adware users face between improved information on consumer products and the violation of their privacy. Given this tradeoff, consumers choose to get adware only if their privacy concerns are small. Otherwise, consumers do not adopt software at all, when only adware is available, or get a commercial software when
there is competition in the market for software.

Our model reveals that when the perceived quality of the software is low, software providers will prefer to distribute it to consumers for free as adware. As the perceived quality of the software increases, software provider will eventually prefer to sell their software commercially. The model also reveals that improvements in the technology of display ads will lead to less violation of privacy and will benefit consumers. By contrast, although a ban on adware will protect the privacy of software users, it will also prevent them from getting targeted information about consumer products and will therefore make them worse off.

Finally, although our model considers the market for software it can also be applicable to media markets. In this context, our model can be used to study the choice of content providers between selling their content to consumers for a fee or distributing it to consumers for free and supporting it with advertising. With this interpretation in mind, our paper is related to some of the literature on media markets. In particular, Hansen and Kyhl (2001) compare two alternatives for financing the broadcasts of special events on TV (like sport events): pay-per-view and TV commercials. They find that advertising leads to a higher consumers’ surplus than pay-per-view. This result is similar to our result that bans on adware (which are akin to bans on advertising in their model) will hurt consumers.

7 References


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